International Journal of Multidisciplinary Research and Technology
ISSN 2582-7359, Peer Reviewed Journal, Impact Factor 6.325
www.ijmrtjournal.com

MATHEMATICAL ANALYSIS AND APPLIED MATHEMATICS USE IN DIFFERENT FIELDS

Dhirendra Kumar Jha
Associate Professor, Department of Mathematics
Ya.La.Na. Bidyapith Matihani

Abstract:

This paper presents the study of Calculus and their applications and Adams- Bash forth - Multon algorithm has been extended to solve delay differential equation fractional order. This report is aimed at the engineering and/or scientific professional who wishes to learn about fractional Calculus and its possible applications in their field(s) of study.

Keywords: Calculus, Laplace, differential equation, Wavelet, dynamic, etc.

Introduction:

The subject of Calculus has applications in diverse and widespread fields of engineering and science such as electromagnetics, viscoelasticity, fluid mechanics, electrochemistry, biological population models, optics, and signals processing. It has been used to model physical and engineering processes that are found to be best described by fractional differential equations. The derivative models are used for accurate modelling of those systems that require accurate modelling of damping. In these fields, various analytical and numerical methods including their applications to new problems have been proposed in recent years. Mathematical modelling of real-life problems usually results in fractional differential equations and various other problems involving special functions of mathematical physics as well as their generalizations in one or more variables. In addition, most physical phenomena of the issue. Accordingly, various papers on fractional differential equations have been included in this special issue after completing a heedful, rigorous, and peer-review process. The issue of robust stability for fractional order Hopfield neural networks with parameter uncertainties is rigorously investigated. Based on the fractional order Lyapunov direct method, the sufficient condition of the existence, uniqueness, and globally robust stability of the equilibrium point is presented. Moreover, the sufficient condition of the robust synchronization between such neural systems with the same parameter uncertainties is proposed owing to the robust stability analysis of its synchronization error system. In addition, for different parameter uncertainties, the quasi-synchronization between the classes of neural networks is investigated with linear control. And the quasi-synchronization error bound can be controlled by choosing the suitable control parameters. Moreover, robust synchronization and quasi-synchronization between the classes of neural networks are discussed. Several wavelet methods such as Haar wavelet method, cubic B-spline wavelet method, Legendre wavelet method, Legendre multiwavelet method, and Chebyshev wavelet method have been examined for solving fractional differential equations. The Legendre multiwavelet method along with Galerkin method can be applied for providing approximate solutions for initial value problems of fractional nonlinear partial differential equations. Using these wavelet methods the fractional differential equations have been reduced to a system of algebraic equations and this system can be easily solved by any usual methods. The distributed coordination of fractional multiagent systems with external disturbances is also discussed. The state observer of fractional dynamical system is presented, and an adaptive pinning controller is designed for a little part of agents in multiagent systems without disturbances. Based on disturbance observers, the controllers are composited with the pinning controller and the state observer. By applying the stability theory of fractional order dynamical systems, the distributed coordination of fractional multiagent systems with external disturbances can be reached asymptotically. Two integral operators involving Appell's functions or Horn's function in the kernel are considered. Composition of such functions with generalized Bessel functions of the first kind is expressed in terms of generalized Wright function and generalized hyper geometric series. The existence of solutions for a nonlinear boundary value problem of impulsive fractional differential equations is studied with - Laplacian operator. The research of boundary value problems for Laplacian equations of fractional order has just begun in recent years.

Most of the mathematical theory applicable to the study of fractional Calculus was developed prior to the turn of the 20th century. However it is in the past 100 years that the most intriguing leaps in engineering and scientific application have been found. The mathematics has in some cases had to change to meet the requirements of physical reality. Caputo reformulated the more 'classic' definition of the Riemann-Liouville fractional derivative in order to use integer order initial conditions to solve his fractional order differential equations [21]. As recently as 1996, Kolowankar reformulated again, the Riemann-Liouville fractional derivative in order to differentiate no-where differentiable fractal functions [22]. Leibniz's response, based on studies over the intervening 300 years, has proven at least half right. It is clear that within the 20th century especially numerous applications and physical manifestations of fractional Calculus have been found. However, these applications and the mathematical background surrounding fractional Calculus are far from paradoxical. While the physical meaning is difficult (arguably impossible) to grasp, the definitions them selves are no more rigorous than those of their integer order counterparts. Due to its applicability in a variety of fields Fractional Calculus (FC) is receiving importance in various branches of Science and Engineering. Unlike ordinary derivative operator, Fractional Derivative Operator (FDO) is non-local in nature. Due to non-local nature of FDO, it can formulate processes having memory and hereditary properties. Fractional calculus is finding applications especially in viscoelasticity, anomalous diffusion process, electro chemistry, fluid flow and so on [4, 17, 18]. Delay Differential Equation (DDE) is a differential equation in which the derivative of the function at any time depends on the solution at previous time. Introduction of delay in the model enriches its dynamics and allows a precise description of the real life phenomena. DDEs are proved useful in control systems [8], lasers, traffic models [3], metal cutting, epidemiology, neuroscience, population dynamics [11], chemical kinetics [7] etc. Even in one dimensional systems interesting phenomena like Chaos are observed (cf. Example 1). In DDE one has to provide history of the system over the delay interval [-_, 0) as the initial condition. Due to this reason delay systems are infinite dimensional in nature. Because of infinite dimensionality the DDEs are difficult to analyse analytically [9] and hence the numerical solutions play an important role. Existence and uniqueness theorems on fractional delay differential equations are discussed in [1, 10, 14, 15]. In this paper we extend the fractional predictor-corrector scheme to solve DDEs of fractional order. Some numerical examples are presented to explain the method.

Discussion:

Fractional Integral Equations :

First kind :

The first form of the fractional integral Equation is given by the form in (1)

$$\frac{1}{\Gamma(\alpha)}\int_0^t \frac{u(\tau)}{(t-\tau)^{1-\alpha}}d\tau = f(t), 0<\alpha < 1 \quad (1)$$

This may also be written as

$$J^{\alpha}u(t) = f(t) \quad (2)$$

The solution of this kind is straight forward, and written

$$\text{u(t)}=D^{\alpha}f(t) \quad (3)$$

One may be tempted to use the right Hand or Caputo definition for the fractional derivative in this situation interchangeably with the LHD, however it must be underscored that not in every situation does $D_*^{\alpha}J^{\alpha}f(t) = f(t)$. In fact, it will be shown below that from a solution arrived at through use of the Laplace transform, a remainder term arises when the RHD is used to solve (1).

In the Laplacian domain, integral equations of the first kind assume the form below :

$$J^{\alpha}u(t) = \Phi_{\alpha}(t) * u(t) \Rightarrow L\{\Phi_{\alpha}(t) * u(t)\} = \frac{\tilde{u}^{(s)}}{s^{\alpha}} \quad (4)$$

Applied Science Periodical [Vol. XV (4), November 13]

Algebraically, we can recorder the result in (4) into one of two forms

$$\tilde{u}(s) = S^{\alpha}\tilde{f}(s) \Rightarrow s\left[\frac{\tilde{f}(s)}{S^{1-\alpha}}\right] \quad (5)$$

or

$$\tilde{u}(s) = S^{\alpha}\tilde{f}(s) \Rightarrow \frac{1}{S^{1-\alpha}}\left[s\tilde{f}(s) - f(o)\right] + \frac{f(0)}{S^{1-\alpha}} \quad (6)$$

Inverting the first form back into the time domain, we get

$$u(t) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha}}d\tau = f(t) \quad (7)$$

which is equivalent to solution of the equation with the LHD. The second form can be similarly inverted to yield

$$u(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t \frac{f\prime(\tau)}{(t-\tau)^{\alpha}}d\tau = f(t) + f(0)\frac{t^{-\alpha}}{\Gamma(1-\alpha)} \quad (8)$$

The first element of this result is the RHD, but as mentioned above, one must include a remainder term that is dependent on the value of the function at 0.

Second Kind :

Integral equations of the second kind follow the form on (9).

$$u(t) + \frac{\lambda}{\Gamma(\alpha)}\int_0^t \frac{u(\tau)}{(t-\tau)^{1-\alpha}}\,d\tau = f(t) \Rightarrow (1 + \lambda J^{\alpha})u(t) = f(t) \quad (9)$$

The solution to (9) is found to be

$$u(t) = (1 - \lambda J^{\alpha})^{-1}f(t) = (1 - \sum_{n=1}^{\infty}(-\lambda)^n J^{\alpha n})f(t) = f(t) + \left(\sum_{n=1}^{\infty}(-\lambda)^n\,\Phi_{\alpha n}\right) * f(t) \quad (10)$$

$$E_{\alpha}(z) = \sum_{k=0}^{\infty}\frac{z^k}{\Gamma(\alpha k+1)}, \alpha > 0.$$

We can show that

$$E_{\alpha}(-\lambda t^{\alpha}) = \sum_{k=0}^{\infty}\frac{(-\lambda\, t^{\alpha})^k}{\Gamma(\alpha k+1)} \quad (11)$$

By taking the first derivative of (12), we eliminate the first term in the $E_{\alpha}(-\lambda t^{\alpha})$ expansion and recover the form of found in (11). Thus, the solution to the integral equation of the second kind can be formally written as

$$u(t) = f(t) + \frac{d}{dt}[E_{\alpha}(-\lambda t^{\alpha})] * f(t) \quad (12)$$

The same solution can be reached by using Laplace Transofmr. Start by taking the Laplace transform of (9).

$$L\{(1+\lambda)^{\alpha}u(t)\} = L\{f(t)\} \rightarrow \left[1 + \frac{\lambda}{s^{\alpha}}\right]\tilde{u}(s) = \tilde{f}(s) \quad (13)$$

Equation (13) can be rearranged in many ways, but one in particular leads us back to the result presented in (12).

$$\tilde{u}(s) = \left[s\frac{s^{\alpha-1}}{s^{\alpha+\lambda}} - 1\right]\tilde{f}(s) + \tilde{f}(s) \quad (14)$$

Equation (14) is next inverted back into the normal function domain. In order to do this one must address comprehend the Laplace transform of a special form of the Mittag Leffler Function, given in (15).

$$L\{E_\alpha(-\lambda t^\alpha)\} = \frac{s^{\alpha-1}}{s^{\alpha+\lambda}} \quad (15)$$

By the relationship given in

$$L\{f^{(x)}(t)\} = s^n \tilde{f}(s) - \sum_{k=0}^{n-1} s^{n-k-1} f^{(k)}(0) = s^n \tilde{f}(s) - \sum_{k=0}^{n-1} s^k f^{(n-k-1)}(0)$$

it is clear that what is between the brackets in the LHS of (14) is simply the Laplace transform of the first derivative of the LHS of (15), i.e.

$$L\{E_\alpha{}^{(1)}(-\lambda t^\alpha)\} = s\frac{s^{\alpha-1}}{s^{\alpha+\lambda}} - 1 \quad (16)$$

From the definition of the Laplace convolution given in

$$f(t) * g(t) := \int_0^t f(t-\tau)g(\tau)d\tau = g(t) * f(t),$$

is easily seen how the inverse of (14) would yield the same result shown in (12).

Definition A : Let $f \epsilon C_\alpha and\ \alpha \geq -1$, then the (left-sided) Riemann-Liouville integral of order $\mu, \mu > 0$ is given by

$$I_t^\mu f(t) = \frac{1}{\Gamma(\mu)} \int_0^t (t-\tau)^{\mu-1} f(\tau)d\tau, \qquad t > 0$$

Definition B : The (left side) cuputo fraction derivative of $f, f \epsilon C_{-1}^m, m \epsilon IN \cup \{0\}$ is define as,

$$D_t^\mu f(t) = \frac{d^m}{dt^m} f(t), \qquad \mu = m$$

$$= I_t^{m-\mu} \frac{d^m f(t)}{dt^m}, m-1 < \mu < m, m\epsilon\ IN$$

Note that for $m-1 < \mu < m, m\epsilon\ IN$

$$I_t^\mu D_t^\mu f(t) = f(t) - \sum_{k=0}^{m-1} \frac{d^k f}{dt^k}(0) \frac{t^k}{k!},$$

$$I_t^\mu t^v = \frac{\Gamma(v+1)}{\Gamma(\mu+v+1)} t^{\mu+v}$$

Example : Consider a fractional order version of the DDE given in

$$D_t^\alpha y(t) = \frac{2y(t-2)}{1+y(t-2)^{9.65}} - y(t), \quad (17)$$

y(t)=0.5, t≤ 0. (18)

We have taken the step size h = 0.01 in this example. Fig. 1(a) shows the solution y(t) of system (17) - (18) for a = 0.97, whereas Fig. 1(b) shows phase portrait of the system i.e. plot of y(t) versus y(t - 2) for the same value of α. It may be observed from these figures that the system shows a periodic (chaotic) behavior. In the following experiments we have decreased the value of a and observed that the system becomes periodic for α<0.87. The periodic behavior of the system can be observed in Fig. 1(c) and 1(d) where we have considered α = 0.85.

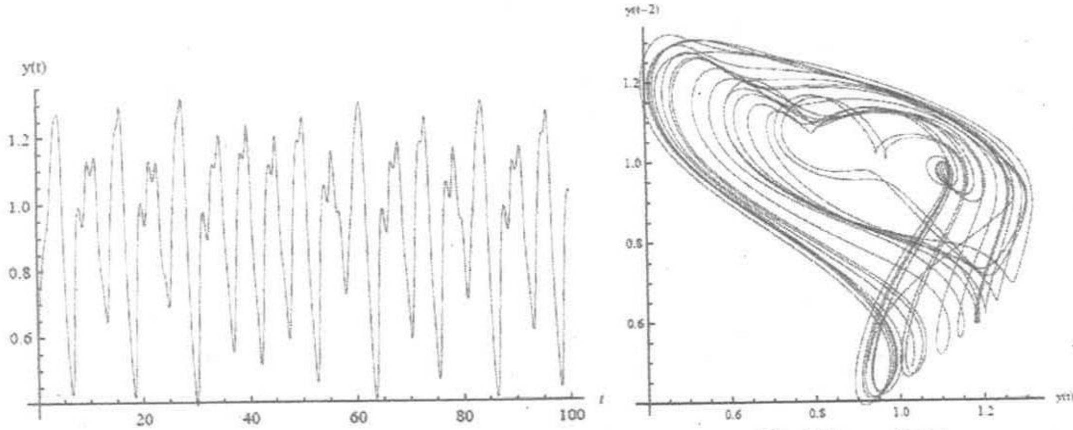

Fig.1(a): $\alpha = 0.97$ Fig.1(b): $\alpha = 0.97$

JFCA-2011/1 A PREDICTOR-CORRECTOR SCHEME 5

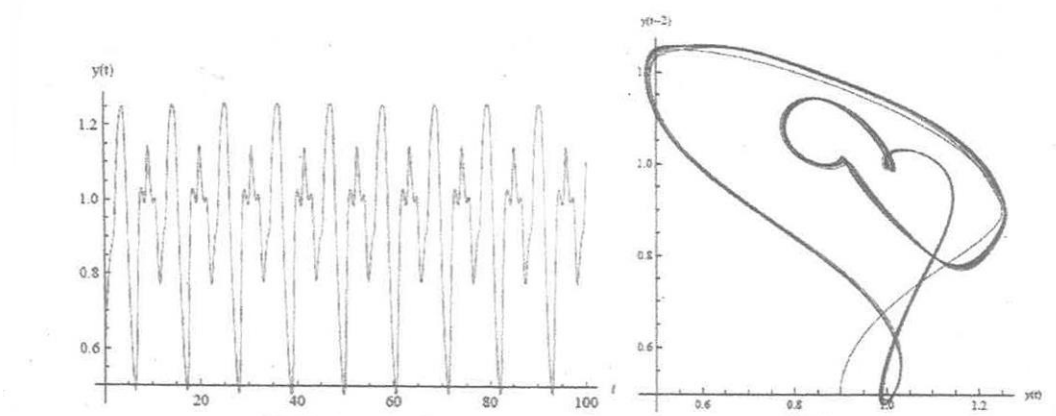

Fig.1(c): $\alpha = 0.85$ Fig.1(d): $\alpha = 0.85$

Conclusion:

There are very few results about Chaos synchronization of the fractional order time delay chaotic systems available in the literature. Here Chaos synchronization of different fractional order time-delay chaotic systems is also considered. Based on the Laplace transform theory, the conditions for achieving synchronization of different fractional order time-delay chaotic systems are analyzed by use of active control technique. The numerical simulations show that the modified equation is more reliable in predicting the movement of pollution in the deform able aquifers than the constant fractional and integer derivatives. At present, the use of fractional order partial differential equation in real-physical systems is commonly encountered in the fields of science and engineering. The efficient computational tools are required for analytical and numerical approximations of such physical models. The present issue has addressed recent trends and developments regarding the analytical and numerical methods that may be used in the fractional order dynamical systems. Eventually, it may be expected that the present special issue would certainly helpful to explore the researchers with their new arising fractional order problems and elevate the efficiency and accuracy of the solution methods for those problems in use now a days.

Interest in Fractional Calculus for many years was purely mathematic, and it is not hard to see why. Only the very basic concepts regarding fractional order Calculus were addressed here, and yet it is evident that the study fractional Calculus opens the mind to entirely new branches of thought. It fills in the gaps of traditional Calculus in ways that as of yet, no one completely understands. Despite the infancy of this field, the small sampling of applicable problems given here are merely a tiny fraction of what is currently being studied. Adams-Bashforth Moulton method is extended to solve fractional differential equations involving delay. Some interesting fractional delay differential equations arising in Biology have been solved. It is observed that even one dimensional delayed systems of fractional order show chaotic behaviour, and below some critical order, the system changes its nature and becomes periodic. In some cases it is observed that the phase portrait gets stretched as the order of the derivative is reduced.

References:

1. M. Benchohra, J. Henderson, S.K. Ntouyas and A. Ouahab: Existence results for fractional order functional differential equations with infinite delay, J. Math. Anal. Appl., 338, 1340- 1350, 2008.
2. W.C. Chen: Nonlinear dynamics and Chaos in a fractional-order financial system, Chaos, Solitons and Fractals, 36, 1305-1314, 2008.
3. L.C. Davis: Modification of the optimal velocity traffic model to include delay due to driver reaction time, Physica A, 319, 557-567, 2002.
4. L. Debnath: Recent applications of fractional Calculus to science and engineering, International Journal of Mathematics and Mathematical Sciences, 2003(54), 3413-3442, 2003.
5. K. Diethelm, N.J. Ford and A.D. Freed: A predictor-corrector approach for the numerical solution of fractional differential equations, Nonlinear Dynamics, 29, 3-22, 2002.
6. K. Diethelm: An algorithm for the numerical solution of differential equations of fractional order, Elec. Trans. Numer. Anal., 5, 1-6, 1997.
7. I. Epstein and Y. Luo: Differential delay equations in chemical kinetics. Nonlinear models: the cross-shaped phase diagram and the Oregonator, J. Chem. Phys., 95, 244-254, 1991.
8. E. Fridman, L. Fridman and E. Shustin: Steady modes in relay control systems with time delay and periodic disturbances, J. Dyn. Sys., Meas., Control, 122, 732-737, 2000.
9. J.K. Hale and S.M.V. Lunel: Introduction to Functional Differential Equations, Applied Mathematical Sciences, No. 99, Berlin, Springer-Verlag, 1993.
10. J. Henderson and A. Ouahab: Fractional functional differential inclusions with finite delay, Nonlin. Anal.70, 2091-2105, 2009.
11. Y. Kuang: Delay Differential Equations with Applications in Population Biology, Academic Press, Boston, San Diego, New York, 1993.
12. C. Li and G. Peng: Chaos in Chen's system with a fractional order, Chaos Solitons Fractals, 22, 443-450, 2004.
13. Y. Luchko and R. Gorenflo: An operational method for solving fractional differential equations with the Caputo derivatives, Acta Math Vietnamica, 24, 207-233, 1999.
14. T.A. Maraaba, F. Jarad and D. Baleanu: On the existence and the uniqueness theorem for fractional differential equations with bounded delay within Caputo derivatives, Sci. China Ser. A 51(10), 1775-1786, 2008.
15. T.A. Maraaba, D. Baleanu and F. Jarad: Existence and uniqueness theorem for a class of delay differential equations with left and right Caputo fractional derivatives, J. Math. Phys. 49(8), [article number: 083507], 2008.
16. M. Okamoto and K. Hayashi: Frequency conversion mechanism in enzymatic feedback systems, J. Theoret. Biol., 108(4), 529-537, 1984.
17. I. Podlubny: Fractional differential equations, Academic Press, 1999.
18. S.G. Samko, A.A. Kilbas and O.I. Marichev: Fractional Integrals and Derivatives: Theory and Applications, Gordon and Breach, Yverdon, 1993.
19. L.J. Sheu, H.K. Chen, J.H. Chen and L.M. Tam: Chaos in a new system with fractional order, Chaos, Solitons and Fractals, 31, 1203-1212, 2007.
20. L. Tavernini: Continuous - Time Modeling and Simulation, Gordon and Breach, Amsterdam, 1996.
21. D.R. Wille and C.T.H. Baker: DELSOL - a numerical code for the solution of systems of delay- differential equations, Appl. Numer. Math., 9, 223-234, 1992.
22. I. Podlubny: Fractional Differential Equations, "Mathematics in Science and Engineering V 198", Academic Press, 1999.
23. K.M. Kowankar & A.D Gangal: Fractional Differentiability of nowhere differentiable functions and dimensions, CHAOS, V. 6, No. 4, 1996, American Institute of Physics.